A Formal Executable Semantics of PROMELA

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Abstract. PROMELA, the modeling language of SPIN, is widely used to specify and model check finite-state concurrent systems but lacks support for deductive verification. This paper presents a faithful, executable semantics of PROMELA in the $\mathbb K$ framework that enables code-level deductive verification. To address the nontrivial interactions between guarded nondeterminism and concurrency, we introduce LOAD-AND-FIRE, an elegant semantic pattern that yields a modular, uniform treatment of guarded nondeterminism, cross-process interference, and atomicity in $\mathbb K$. Our semantics enables the full suite of analyses provided by $\mathbb K$, including deductive verification of PROMELA programs with infinite state spaces, a capability previously unavailable for PROMELA models. We illustrate the approach with a case study in deductive verification of an infinite-state concurrent system.

1 Introduction

PROMELA (PROcess MEta LAnguage) [1], the modeling language of the SPIN model checker, has been widely used for decades to specify concurrent systems. It combines imperative constructs—such as assignments, conditionals, and loops—with first-class nondeterministic choice and message passing via buffered and rendezvous channels. PROMELA is broadly adopted across academia and industry for teaching, prototyping, and validating concurrency designs in domains including protocols, operating systems, and multithreaded software [16,5,10].

Given PROMELA's widespread use, many verification tasks increasingly demand deductive reasoning: parametric invariants (e.g., "for all buffer capacities N"), proofs for infinite-state systems, compositional arguments, and reusable proof artifacts. SPIN's explicit-state LTL model checking [13] is highly effective for analyzing finite-state concurrent systems, yet it does not directly address these needs. What is missing is a mechanized, executable semantics of PROMELA that provides both a precise, runnable reference for the language and a basis for code-level deductive verification.

Prior semantic definitions of PROMELA, such as structural operational semantics [38,17], labeled transition systems [2,24], and denotational semantics [18], provide insight but typically lack mechanized formalizations that support deductive verification of PROMELA programs. SPIN remains the reference implementation, yet it is informally specified [23]. To our knowledge, no prior line of work offers, in a single framework, (i) an executable interpreter for PROMELA; (ii) a proof system that reasons about PROMELA programs; and (iii) modular extensibility that scales to the language's difficult features.

In this paper, we present a faithful and executable semantics of PROMELA in \mathbb{K} that also supports deductive verification. \mathbb{K} [30] is a rewriting-based semantic framework that enables modular, machine-readable definitions of programming languages and automatically derives tools such as interpreters, model checkers, and deductive verifiers from the semantics. Concretely, our semantics (i) runs PROMELA models as an interpreter, (ii) provides a proof engine (kprove) to establish safety properties for both finite-state and infinite-state models, and (iii) constitutes a precise, machine-readable reference for the language.

Defining a faithful, executable semantics for PROMELA is challenging because the language exhibits nontrivial interactions between guarded nondeterminism and concurrency. For example, PROMELA permits nondeterministic choice whose options are guarded by arbitrary statements rather than Boolean conditions, and those statements may themselves nest additional choices. A satisfactory semantics must surface enabledness at the frontier before committing to a step and integrate blocking, nondeterminism, and atomicity in a uniform way. Formalizing these interactions in an elegant, executable form is difficult; consequently, prior work on PROMELA often targets restricted subsets that avoid these complications.

To address these challenges, we propose the Load-and-fire semantic pattern. Load normalizes nested guarded constructs (including statement-guarded options) into a canonical multiset of enabled frontiers and is purely structural and produces no effects. Fire then commits atomically to one enabled frontier. A simple lock mechanism enforces atomic blocks, and a lock-tossing rule preserves atomicity across rendezvous (e.g., a send inside an atomic block transfers the lock to the receiver). This pattern yields modular rules and a uniform treatment of nested guarded nondeterminism, cross-process interference, and atomicity.

Our semantics enables the full suite of analyses provided by the \mathbb{K} framework [15], including deductive verification of PROMELA programs with infinite state spaces, a capability previously unavailable for PROMELA models. It is worth noting that the executability of our semantics enables *validation* by differential execution against the reference implementation, SPIN [13]. We use benchmark examples—including those from the SPIN repository—as a conformance suite, also covering corner cases that exercise the nontrivial features described above.

The contributions of this paper are summarized as follows. (1) We present the first mechanized, executable semantics of PROMELA in \mathbb{K} that enables code-level deductive verification. (2) We introduce the LOAD-AND-FIRE semantic pattern for guarded nondeterminism with arbitrary statement guards, applicable to languages with similar features (e.g., Go's select). We demonstrate the approach with a case study in deductive verification of an infinite-state concurrent system.

The rest of the paper is organized as follows. Section 2 provides background on the \mathbb{K} framework and PROMELA. For readability, we present the semantics incrementally: Section 3 considers a core subset without nested guards, atomic blocks, or impure expressions; Section 4 extends this subset with nested guards; and Section 5 adds atomic blocks and impure expressions. Section 6 presents a case study in deductive verification. Section 7 discusses related work. Finally, Section 8 presents some concluding remarks.

2 Preliminaries

2.1 The \mathbb{K} Semantics Framework

 \mathbb{K} [30] is a semantic framework for programming languages, based on rewriting logic [19]. It has been widely used to formalize a variety of languages, including C [9], Java [3], JavaScript [25], and so on. \mathbb{K} provides several analysis tools, such as deductive verifiers and symbolic execution engines.

Semantics Definition in \mathbb{K} . In \mathbb{K} , program states are represented as multisets of nested cells, called *configurations*. Each cell represents a component of a program state, such as computations and stores. Transitions between configurations are specified as (labeled) \mathbb{K} rules, specifying only the relevant parts of these cells.

A computation in \mathbb{K} (called the \mathbb{K} continuation) is defined as a \curvearrowright -separated sequence (of sort K) of computational tasks (of sort KItem). For example, $t_1 \curvearrowright t_2 \curvearrowright \ldots \curvearrowright t_n$ represents the computation consisting of t_1 followed by t_2 , and so on. A task can be decomposed into simpler tasks, and the result of a task is forwarded to the subsequent tasks. E.g., (5+x)*2 is decomposed into $x \curvearrowright 5+\square \curvearrowright \square*2$, where \square is a placeholder for the result of a previous task. If x evaluates to some value, say 4, then $4 \curvearrowright 5+\square \curvearrowright \square*2$ becomes $5+4 \curvearrowright \square*2$, which eventually becomes 18. We denote the empty task as "K".

The following shows a typical example of $\mathbb K$ rules for variable lookup, where lookup is a label, the k cell contains a computation, env contains a map from variables to locations, and store contains a map from locations to values:

$${\tt lookup:} \ \frac{\langle x \curvearrowright \ldots \rangle_k \ \langle \ldots x \mapsto l \ldots \rangle_{env} \ \langle \ldots l \mapsto v \ldots \rangle_{store}$$

A horizontal line represents a state change, and "..." indicates irrelevant parts. A cell without horizontal lines is not changed by the rule. By the lookup rule, if the first item in k is x, then x is replaced by the value v of x in its location l.

 \mathbb{K} is effective at defining nontrivial features of real-world languages regarding control flow, concurrency, and nondeterminism. This allowed many features in C (e.g., pointers, goto, malloc) to be defined near completely in \mathbb{K} [9]. Likewise, \mathbb{K} 's inherent concurrency and nondeterminism enabled elegant treatments of Java multithreading [3] and JavaScript for-in enumeration [25].

For example, goto statements can be straightforwardly defined in \mathbb{K} , by treating control as explicit data. A standard approach [9] is to preprocess a gotoMap cell containing a map from labels to continuations, encoding the local control-flow for each label. The following shows a simple \mathbb{K} rule for goto statements:

$$\mathtt{goto} \colon \frac{\langle \mathtt{goto} \ X \ \curvearrowright \ldots \rangle_k \ \langle \ldots X \mapsto K \ldots \rangle_{gotoMap}}{K}$$

This rules matches the label X from the first item $goto\ X$ in the k cell, looks up for the continuation K bound to X in gotoMap, and then replaces the current continuation by K.

```
requires "IMP.k"
                                                              <store>
   (\ldots)
                                                              s |-> (
3
   module SPEC
                                                                S:Int
     imports IMP // IMP semantics
                                                     13
                                                                S +Int ((N +Int 1) *Int N /Int 2)):
                                                     14
                                                              n \rightarrow (N:Int => 0)
6
     claim
       \langle k \rangle while (0 < n)
                                                     16
                                                              </store>
                                                            requires N >=Int 0
           { s = s + n; n = n - 1; }
       => .K ...</k>
                                                          endmodule
```

Fig. 1: An example reachability claim in spec.k.

Deductive Verification in \mathbb{K} **.** The \mathbb{K} framework provides a language-agnostic deductive verifier based on Reachability Logic (RL) [6], which is parameterized by the user-defined \mathbb{K} semantics of a language. It has been widely used to verify real-world programs, such as smart contracts [11,27,26].

RL is a proof system for proving partial correctness of a program, whose properties are specified as (all-path) reachability claims of the form $\phi \Rightarrow \psi$, where ϕ and ψ are Matching Logic [29] patterns describing a set of states (i.e., \mathbb{K} configurations). It means: any terminating path from a state satisfying ϕ reaches a state satisfying ψ . Skeirik et al. [32] extended the use of reachability claims as partial correctness to specify invariant properties of nonterminating systems, by "freezing" every reachable state of a nonterminating path to induce a corresponding finite prefix for which reachability claims apply.

Figure 1 shows a concrete example of a reachability claim, excerpted from \mathbb{K} 's tutorial¹. \mathbb{K} configurations in the claim are written in an xml-like notation, where the reachability relation "=>" is written locally within a cell to denote the difference between the initial and goal patterns of the claim. The claim asserts: upon termination of the while loop (line 9), the initial value of \mathbf{s} is incremented by ((N+1)*N/2) (line 14), assuming the initial value of \mathbf{n} is N>0 (line 17).

This claim can be proved automatically via the following process: first, one writes the claim in the file spec.k, importing the semantics definition (e.g., imp.k). Then, running kprove for spec.k completes the proof as follows:

```
$ kprove spec.k
(output) #Top // success
```

Typically, kprove may require auxilary claims to prove the main claim $\phi \Rightarrow \psi$. Auxilary claims can be used to ignore the cycles reachable from ϕ , as they correspond to infinite paths. When such cycles involve an intermediate pattern ϕ' but not ϕ , they should be discharged via the auxilary claim $\phi' \Rightarrow \psi$.

2.2 The PROMELA Language

PROMELA is a popular high-level modeling language for specifying concurrent and distributed systems. It is used as the input language for the SPIN [13]

¹ https://kframework.org/k-distribution/k-tutorial/1_basic/22_proofs/

```
::= Stmt^+
                                                           CStmt ::= if \ Option^+ \ fi
Seq
                                                                      | do Option^+ od
Stmt ::= BStmt \mid CStmt \mid Decl
       ::= Type \ Id[Int]
                                                                       goto Id \mid break
          | \operatorname{chan} Id[Int] = [Int] \text{ of } \{ Type^+ \}
                                                                       atomic { Seq }
Type ::= int \mid bool \mid chan
                                                           Expr
                                                                  := run \ Id(Expr^*)
BStmt ::= Expr
                                                                       Id[Expr] \mid Int \mid Bool
           | Id[Expr] = Expr
                                                                        nfull(Id[Expr])
           Id[Expr] ! Expr^{+}
                                                                        Id[Expr] ? [Arg^+]
          \mid Id[Expr] ? Arg^+
                                                                       | \ominus Expr | Expr \odot Expr
       ::= Id[Expr] \mid Int \mid Bool
                                                           Option ::= :: Expr \rightarrow Seq
```

Fig. 2: An abstract syntactic subset of the full PROMELA. +/* denote repetitions in EBNF notation; unary/binary operators are denoted \bigcirc/\bigcirc , resp.

model checker—which received the 2001 ACM Software System Award [1]—for verification and simulation. SPIN/PROMELA has been applied to a wide range of systems, including cryptographic protocols [16], Linux synchronization primitives [10], and flight-guidance systems [5].

A PROMELA program consists of declarations of global data (e.g., integers, booleans, channels) and processes (proctype). Like C functions, process declarations provide arguments and a body containing a ;-separated sequence (Seq) of statements, whose syntax considered in this paper is given in Figure 2. Statements include basic statements (BStmt) which define one-step atomic action, and control statements (CStmt) which organize the control-flow. All variables are considered as arrays; e.g., a variable x is syntactic sugar for x[0] (unlike C). An informal description of PROMELA is available on SPIN's official website².

Basic statements include condition statements, assignments, and channel operations. A condition statement is a standalone expression that gets skipped when it evaluates to true. It is typically used to guard the subsequent sequence. Channel operations are send/receive operations through a channel.

Similar to related formalisms (e.g., Hoare's CSP [12], Dijkstra's GCL [8]), PROMELA provides features for nondeterminism with if/do statements, whose inner options (*Option*) are selected nondeterministically when enabled. Unlike CSP/GCL, there is no syntactic restriction for the guards in each option sequence; options may start with arbitrary statements, allowing nested options.

A PROMELA program models a concurrent system of processes instantiated from proctype declarations. Processes may be active at startup (by annotating the proctype with active) or spawned by run expressions (e.g., in the initializer process init). Concurrent processes execute as interleavings of locally executed basic statements. Enclosing a sequence SL in atomic $\{SL\}$ enforces atomic execution. Processes communicate via global variables or channels. Channels are either buffered or handshake: buffered channels provide asynchronous FIFO queues of finite capacity, while handshake channels synchronize sender and

² https://spinroot.com

Fig. 3: A producer and consumer model written in PROMELA.

receiver. A channel c is declared as chan c = [N] of $\{TL\}$, where N is the capacity (N = 0 for handshake) and TL specifies the message format.

In PROMELA, basic statements can only be executed if their enabledness (or executablility in PROMELA term) condition holds; otherwise, they block until they become enabled by global state changes. For example, the buffered receive c? x is enabled iff the channel c is nonempty and the arguments match the message. Likewise, a sequence of statements is enabled iff its first statement is enabled; an atomic statement is enabled iff its inner sequence is enabled; an if/do statement is enabled iff either one of the options is enabled. Accordingly, atomic acquires atomicity only when the inner sequence is enabled; if/do chooses an option when it is enabled.

Figure 3 shows a producer-consumer model written in PROMELA. Two active processes, producer and consumer, run concurrently and communicate via the buffered channel c. In its do loop, producer either (when turn == 0) sends 42 on c and sets turn to 1 (line 5), or (when turn != 0) executes skip (line 6). consumer behaves symmetrically: when turn == 1 it receives 42 from c into x and sets turn to 0 (line 11). Here, "->" and skip are syntactic sugar for; and true (as a condition statement), respectively.

3 A Basic Semantics of PROMELA in \mathbb{K}

In this section, we give an overview of the semantics of PROMELA in \mathbb{K} . Rather than covering the full syntax presented in Figure 2, we focus on a simplified subset for which it is straightforward to define \mathbb{K} rules. In subsequent sections, we use these rules as a baseline and extend them modularly to cover the full syntax within our scope.

3.1 Syntactic Subset

We impose three syntactic restrictions on the full syntax shown in Figure 2.

(EXPRESSION-GUARDED OPTIONS) We require that each nondeterministic option in if/do start with an explicit expression as its guard. Namely, we restrict the syntax of *Option* to be:

 $Option ::= :: Expr \rightarrow Seq$

```
 \langle \langle \langle Id \rangle_{ptName} \ \langle List\{Decl\} \rangle_{params} \ \langle Seq \rangle_{code} \ \langle Id \hookrightarrow K \rangle_{gotoMap} \rangle_{ptype^*} \rangle_{ptypes} \\ \langle \langle \langle Int \rangle_{pid} \ \langle Id \rangle_{pName} \ \langle K \rangle_k \ \langle Id \times Int \hookrightarrow Ref \rangle_{env} \ \langle Set\{Id\} \rangle_{lVars} \rangle_{proc^*} \rangle_{procs} \\ \langle Int \hookrightarrow PVal \rangle_{str} \ \langle Int \hookrightarrow Queue \rangle_{net} \ \langle Int_{\perp} \rangle_{lock} \\ \langle Int \rangle_{nextPid} \ \langle Int \rangle_{nextLoc} \ \langle Int \rangle_{nextBCid} \ \langle Int \rangle_{nextHCid} \\ BChan ::= bch(Int) \qquad Value ::= PVal \ | CVal \\ HChan ::= bch(Int) \qquad Ref ::= loc(Int) \ | CVal \\ CVal ::= uch \ | BChan \ | HChan \qquad Queue ::= q(Int, Msg^*) \\ PVal ::= Int \ | Bool \qquad Msg ::= m(Value^*)
```

Fig. 4: (a) Top-Level Configuration (b) Semantic Domains

Under this restriction, the enabledness conditions of options are explicitly tied to the outer if/do wrappers, so that nondeterministic selection is defined in a straightforward way: an option can be selected if its guard evaluates to true. This restriction will be dropped in Section 4.

(BASIC GRANULARITY) We assume a simple setting in which basic statements are the sole granularity of atomic execution in PROMELA: interleaving *cannot* occur during the execution of a basic statement, and *can* occur whenever a basic statement completes. This is enforced by excluding atomic statements from the syntactic category *CStmt*. This restriction will be dropped in Section 5.

(Pure Expressions) We assume that every expression used in PROMELA code is pure: evaluating an expression never produces side effects. To enforce this, we exclude run expressions from the syntactic category Expr, where expressions containing run as a sub-expression carry the side effect of spawning a process. This restriction will be dropped in Section 5.

3.2 Semantic Domains and Configuration

We describe the structure of our configuration in Figure 4a upon which the \mathbb{K} rules are defined. Some core semantic domains are shown in Figure 4b.

The configuration contains core semantic components as nested cells such as: a set of proctype declarations (ptypes); a set of active processes (procs); a store (store) mapping locations to primitive values (PVal); a network (net) mapping buffered channel ids to the corresponding queues (Queue) of messages (Msg).

A ptype cell contains static informations of a process: e.g., its name (ptName), parameters (params), code (code), and the gotomap (gotoMap). A proc cell contains dynamic informations of a process: e.g., its continuation (k), pid (pid), local variables (lVars), and environment (env). env cell contains a mapping from integer-indexed variables (e.g., x[0]) to a reference (Ref), either to a store location, or to a buffered channel. The lock cell contains a global lock used for enforcing atomic execution of a given process; it may contain a pid to which exclusive execution is granted, or \bot indicating that the lock is free. The cells nextPid, nextLoc, nextBCid, and nextHCid are used to supply fresh identifiers.

$$\left\langle \begin{array}{l} \langle 0 \rangle_{pid} \ \langle \mathrm{producer} \rangle_{pName} \ \langle \mathrm{turn} = 1 \frown ... \rangle_k \ \langle \emptyset \rangle_{IVars} \\ \langle \mathrm{x} [0] \mapsto loc(\theta); \mathrm{turn} [0] \mapsto loc(1); \mathrm{c} [0] \mapsto bch(\theta) \rangle_{env} \\ \langle 1 \rangle_{pid} \ \langle \mathrm{consumer} \rangle_{pName} \ \langle \mathrm{do} \ ... \ \mathrm{od} \rangle_k \ \langle \emptyset \rangle_{IVars} \\ \langle \mathrm{x} [0] \mapsto loc(\theta); \mathrm{turn} [0] \mapsto loc(1); \mathrm{c} [0] \mapsto bch(\theta) \rangle_{env} \\ \rangle_{proc} \\ \langle \langle (\cdots) \rangle_{ptype} \langle (\cdots) \rangle_{ptype} \rangle_{ptypes} \langle 0 \mapsto 0; 1 \mapsto 0 \rangle_{str} \ \langle 0 \mapsto queue(10, m(42)) \rangle_{ne} \\ \langle \perp \rangle_{lock} \ \langle 2 \rangle_{nextPid} \ \langle 2 \rangle_{nextLoc} \ \langle 1 \rangle_{nextBCid} \ \langle 0 \rangle_{nextHCid}$$

Fig. 5: A K configuration of the producer-consumer model

Figure 5 shows the \mathbb{K} configuration of the produce-consumer model from Section 2, after executing two basic statements turn == 0; c ! 42 in producer. Note that the global variables x and turn point to the values stored in str, and the channel c points to the queue (capacity 10) with a message 42 in net.

Following the notational conventions of \mathbb{K} , we omit nested structure when unambiguous: e.g., when k and env cells appear in \mathbb{K} rules without mentioning any proc, they are implicitly assumed to be contained in the same proc cell.

3.3 Basic Statements Under Concurrency

As noted, basic statements should execute atomically. Ideally, we can enforce this in the \mathbb{K} framework by encoding each complete one-step behavior of basic statements as a $single~\mathbb{K}$ rule. This would immediately enforce atomic execution for each basic statement.

However, because PROMELA is a high-level language, the behavior of basic statements often decomposes into several orthogonal steps that are more modular to specify as separate rules. For example, a buffered receive c? x, y naturally expands into: "enabledness check; dequeue; assign x; assign y", where \mathbb{K} rules are modularly defined for each of these intermediate steps.

Under concurrency, this modular style raises consistency concerns. For instance, if two processes p1 and p2 execute c? x, y concurrently when the channel c holds a single message, an interleaving such as "enabledness check (p1); enabledness check (p2); dequeue (p1)" would abruptly block p2 from dequeueing the message.

To prevent such inconsistencies while retaining a modular style, we introduce two special \mathbb{K} rules, fire and rel, which enforce mutual exclusion between concurrent applications of execution rules performing intermediate computations of basic statements. Informally, for a basic statement BS, fire initiates the execution of BS by granting a free lock to the owner process, upon checking the enabledness of BS. rel then completes the execution by releasing the lock. The following rules formalize this behavior:

$$\text{fire: } \frac{\langle P \rangle_{pid}}{effect(BS) \curvearrowright \texttt{\#rel}} \curvearrowright \ldots \rangle_k \stackrel{\langle \bot \rangle_{lock}}{\overline{P}} \langle ENV \rangle_{env} \langle STR \rangle_{str} \langle NET \rangle_{net}}{\overline{P}}$$

$$\text{if } \llbracket enabled(BS) \rrbracket_{ENV,STR,NET} = \top$$

$$\text{rel: } \frac{\langle \texttt{\#rel}}{K} \curvearrowright \ldots \rangle_k \stackrel{\langle \bot \rangle_{lock}}{\overline{\bot}}$$

Here, effect(BS) is defined to be a \mathbb{K} continuation representing intermediate computations for BS, while enabled(BS) denotes the enabledness condition. fire rule checks enabled(BS) as a side-condition³, via evaluation function $\llbracket \cdot \rrbracket_{(\cdot,\cdot,\cdot)}$ parameterized by the local environment, the store, and the network. Note that by BASIC GRANULARITY, the lock is released immediatly after effect(BS) completes.

The side-effects and the enabledness conditions for the four basic statements—namely, condition/assignment/buffered send/buffered receive— are defined as follows, via the functions $effect: BStmt \rightarrow K$ and $enabled: BStmt \rightarrow Expr$.

```
\begin{array}{ll} effect(E) = .K & enabled(E) = E \\ effect(X \llbracket E \rrbracket = E') = \# \operatorname{ssign}(X, E, E') & enabled(X \llbracket E \rrbracket = E') = true \\ effect(X \llbracket E \rrbracket \ ! \ EL) = \# \operatorname{send}(X, E, EL) & enabled(X \llbracket E \rrbracket \ ! \ EL) = n full(X \llbracket E \rrbracket) \\ effect(X \llbracket E \rrbracket \ ? \ AL) = \# \operatorname{recv}(X, E, AL) & enabled(X \llbracket E \rrbracket \ ? \ AL) = X \llbracket E \rrbracket ? \llbracket AL \rrbracket \end{array}
```

Here, the side-effects are represented by *effect markers* (e.g., #assign), which is reduced via the (multi-step) execution rules, *atomically*. Handshake operations are excluded here; they are handled specially via a dedicated firing rule below.

Example (Assignment). We demonstrate concretely how fire and rel help define execution rules for assignments in a modular way. The following rules reduce the effect marker #assign(X, E, E') in two steps:

$$\text{assign-eval:} \begin{array}{c} \langle \texttt{\#assign}(X, \underbrace{E}, \underbrace{E'}) \land \ldots \rangle_k \\ \hline { \llbracket E \rrbracket_{ENV,STR,NET}} \end{array} \\ \langle ENV \rangle_{env} \, \langle STR \rangle_{str} \, \langle NET \rangle_{net} & \text{if at least one of E or E' is a non-value} \\ \\ \text{assign-prim:} & \langle \texttt{\#assign}(X,I:Int,V:PVal) \land \ldots \rangle_k \, \langle \ldots X[I] \mapsto loc(J) \ldots \rangle_{env} \, \langle \ldots J \mapsto \underbrace{\ldots} \rangle_{str} \\ \\ \text{assign-chan:} & \langle \texttt{\#assign}(X,I:Int,V:CVal) \land \ldots \rangle_k \, \langle \ldots X[I] \mapsto \underbrace{\ldots} \rangle_{env} \\ \end{array}$$

The assign-eval rule evaluates the index E to an integer I and E' to a value V. If V is a primitive value, assign-prim updates the store by V at the location pointed by X[]; otherwise, if V is a channel value, assign-chan updates the environment so that X[I] points to the channel V.

Now consider the assignment x = x + y in the process with pid 1, where x = 4 and y = 2. This executes in lock-step via the sequence "fire; assign-eval;

³ In the paper, we use \top/\bot and true/false interchangeably for boolean values.

assign-prim; rel", as shown by the trace below (mutated portions are underlined):

```
\begin{split} &\langle 1 \rangle_{pid} \ \langle \underline{\mathbf{x}} = \underline{\mathbf{x}} + \underline{\mathbf{y}} \curvearrowright K \rangle_k \ \langle \underline{\mathbf{x}}[0] \mapsto 0; \underline{\mathbf{y}}[0] \mapsto 1 \rangle_{env} \ \langle 0 \mapsto 4; 1 \mapsto 2 \rangle_{str} \ \langle \underline{\perp} \rangle_{lock} \\ &\rightarrow \langle 1 \rangle_{pid} \ \langle \mathtt{\#assign}(\underline{\mathbf{x}}, \ 0, \ \underline{\mathbf{x}} + \underline{\mathbf{y}}) \curvearrowright \mathtt{\#rel} \curvearrowright K \rangle_k \ \langle \underline{\mathbf{x}}[0] \mapsto 0; \underline{\mathbf{y}}[0] \mapsto 1 \rangle_{env} \ \langle 0 \mapsto 4; 1 \mapsto 2 \rangle_{str} \ \langle 1 \rangle_{lock} \\ &\rightarrow \langle 1 \rangle_{pid} \ \langle \mathtt{\#assign}(\underline{\mathbf{x}}, \ 0, \ \underline{6}) \curvearrowright \mathtt{\#rel} \curvearrowright K \rangle_k \ \langle \underline{\mathbf{x}}[0] \mapsto 0; \underline{\mathbf{y}}[0] \mapsto 1 \rangle_{env} \ \langle 0 \mapsto \underline{4}; 1 \mapsto 2 \rangle_{str} \ \langle 1 \rangle_{lock} \\ &\rightarrow \langle 1 \rangle_{pid} \ \langle \mathtt{\#rel} \curvearrowright K \rangle_k \ \langle \underline{\mathbf{x}}[0] \mapsto 0; \underline{\mathbf{y}}[0] \mapsto 1 \rangle_{env} \ \langle 0 \mapsto 6; 1 \mapsto 2 \rangle_{str} \ \langle \underline{1} \rangle_{lock} \\ &\rightarrow \langle 1 \rangle_{pid} \ \langle K \rangle_k \ \langle \underline{\mathbf{x}}[0] \mapsto 0; \underline{\mathbf{y}}[0] \mapsto 1 \rangle_{env} \ \langle 0 \mapsto 6; 1 \mapsto 2 \rangle_{str} \ \langle \underline{1} \rangle_{lock} \end{split}
```

3.4 Communication via Channels

Following the approach above, we present the execution rules for channel operations. This concludes our execution rules for all four basic statements; condition statements do not require any execution rules by Pure Expressions.

Buffered Channels. Communication via a buffered channel proceeds by processing the markers #send(X, E, EL) and #recv(X, E, AL), where $EL : Expr^*$ and $AL : Arg^*$ are sender's massage payloads and receiver's arguments, respectively. We list the execution rules for buffered channels below.

The rules send-eval and recv-eval work similarly to assign-eval. By slight abuse of notation in send-eval, the evaluation function $[\![\cdot]\!]_{(\cdot,\cdot,\cdot)}$ is used as a pointwise extension, returning $Value^*$ from $Expr^*$. send-msg enqueues the message m(VL) to the queue pointed by X_I (through a buffered channel bch(C)). In a similar way, recv-msg dequeues the message from the queue, and then receives the message to the arguments by iterative assignments via the rules assign-msg1 – assign-msg3.

Handshake Channels. As mentioned, the firing rule for handshake is separately defined, as it involves two k cells for inter-process synchronization:

$$\begin{array}{l} \text{hs-fire:} & \langle ... \langle X \, [E] \, \mid \, EL \, \smallfrown ... \rangle_k \, \, \langle ENV \rangle_{env} \, ... \rangle_{proc} \, \, \langle STR \rangle_{str} \, \, \langle NET \rangle_{net} \, \, \langle \bot \rangle_{lock} \\ & \langle ... \langle P \rangle_{pid} \, \langle \underbrace{X' \, [E'] \, ? \, AL}_{\text{\#assignMsg}(AL, EL) \, \curvearrowright \, \#rel} \, \, \, ... \rangle_k \, \, \langle ENV' \rangle_{env} \, ... \rangle_{proc} \\ & \text{if} \, \, [\![X \, [E]]\!]_{ENV,STR,NET} =_{HChan} \, [\![X' \, [E']]\!]_{ENV',STR,NET} \, \, \wedge \, \, AL = [\![EL]\!]_{ENV,STR,NET} \end{array}$$

The side condition checks if X[E] and X'[E'] denote the same handshake channel and if the sender's payload EL matches the receiver arguments AL. After handshake, the control is given to the receiver (P), who continues to receive the message via assign-msg1 – assign-msg3 defined above.

3.5 Other \mathbb{K} Rules

We list other \mathbb{K} rules that are worth mentioning. The rules for local variable declaration can be found in Appendix A.

Nondeterminism. As Expression-Guarded Options require each option's enabledness condition to appear as its guard, the nondeterministic selection rule (select) for if statements can be defined straightforwardly, which selects the option whose guard evaluates to true and yields its sequence.

$$\text{select: } \frac{\langle \text{if } OL \ (:: E \rightarrow SL) \ OL' \ \text{fi}}{SL} \curvearrowright ... \rangle_k \ \langle ENV \rangle_{env} \ \langle STR \rangle_{str} \ \langle NET \rangle_{net} \ \langle \bot \rangle_{lock}$$

$$\text{if } \llbracket E \rrbracket_{ENV,STR,NET} = \top$$

Note that this rule requires the global lock to be free, thereby preventing inconsistent guard evaluation during the execution of a basic statement.

Structural Rules. Below we list the structural rules, which rearrange the configuration or control without changing the program's observable state.

The seq rule decomposes a sequence SL into \mathbb{K} continuations. The goto rule performs an unconditional jump to the destination label, as covered in Section 2. We assume gotoMap is appropriately preprocessed, eliminating all the labels in the code. The loop rule produces if indefinitely, whenever do is encountered.

```
Fig. 6: Nested Options
```

Fig. 7: Cross-Process Interference

The loop breaks by the rules break1 and break2 (the side-condition "otherwise" means no other rules match).

4 Handling Nondeterminism

In this section, we drop the Expression-Guarded Options restriction introduced in the previous section. This syntactic extension allows options to be guarded by arbitrary statements, as in Figure 2. Consequently, the previous select rule no longer applies under the extended syntax. Devising a new set of rules for this setting is nontrivial. We begin with motivating examples that lead to our Load-And-Fire approach, which we develop at the end of this section.

In Figure 6, the outer if involves nested options: the second option is guarded by a whole do block. Consequently, if is guarded by the three *leading* basic statements A, B, and C, each of which can be selected when enabled.

In Figure 7, both of the if statements in processes p1, p2 are guarded by send/receive operations via a handshake channel c. This raises cross-process interference among two nondeterministic choices: selecting the first option in p1 forces p2 to take its second (the handshake must occur), whereas selecting the second option in p1 forces p2 to take its first (the handshake cannot occur).

As motivated by the examples, if statements may involve (i) nested options, (ii) cross-process interference, or (iii) their combinations. Consequently, unlike select, their behavior cannot be specified by simply matching a guard (i.e., a leading basic statement) among the options, since guards may appear at arbitrary nesting depth within options.

4.1 The Load-and-Fire Approach

We address the challenge above by introducing loading rules, which are structural normalization rules that flatten nested if options into a canonical form. The canonical form is a multiset of \mathbb{K} continuations, each being the flattened computation for each local branch of if.

For example, a continuation of the form if ... fi $\land K_{global}$ is normalized by loading rules, into the canonical form $[BS_1 \land K_1 | \cdots | BS_n \land K_n] \land K_{global}$ for each leading basic statement BS_i appearing in if ... fi, where $[_|_]$ is the multiset constructor. Intuitively, this canonical form represents a "superposition" of local continuations $BS_i \land K_i \land K_{global}$, where each BS_i is available for

syntactic matching (modulo associativity/commutativity for multisets) at the top-level of the continuation.

Accordingly, we can lift the previous fire and hs-fire rules to match under this set-lifted context, organized by the loading rules. This allows the firing rules to match against a basic statement under arbitrarily nested options. Below we give concrete definitions of the (lifted) firing rules and the loading rules, and revisit the motivating examples to demonstrate how they work.

Firing Rules. The following rules are the lifted versions of fire and hs-fire:

Compared to the previous versions, the changes are found only in the k cells: A basic statement BS is matched under the set-lifted context $[BS \curvearrowright K_{local} \mid K_{rest}]$, where K_{rest} matches the other branches. When BS fires, the set-lifted continuation collapses back into a linear form, reflecting a nondeterministic selection.

Loading rules. Loading rules decompose a continuation of the form if ... fi \curvearrowright K into $[BS_1 \curvearrowright K_1 \mid \cdots \mid BS_m \curvearrowright K_n] \curvearrowright K$, so that enabledness becomes visible at the front of each branch. As a special case, $BS \curvearrowright K$ is also lifted to $[BS] \curvearrowright K$, viewing $BS \equiv \text{if} :: BS$ fi. The loading rules are defined as follows:

$$\begin{aligned} & \operatorname{load-lift:} \langle \underbrace{S}_{\# \operatorname{load}} \curvearrowright [S] \curvearrowright \ldots \rangle_k & \text{if } Loadable(S) = \top & \operatorname{load-bs:} \frac{\# \operatorname{load}}{.K} \curvearrowright [BS \curvearrowright K] \\ & \operatorname{load-seq:} LT \curvearrowright [\underbrace{S; SL}_{S \curvearrowright SL} \curvearrowright K] & \operatorname{load-do:} LT \curvearrowright [\underbrace{.K}_{\operatorname{if}} \curvearrowright \operatorname{do} OL \operatorname{od} \curvearrowright K] \\ & \operatorname{load-if1:} \underbrace{LT \curvearrowright [\operatorname{if} \ (:: SL) \ OL \ \operatorname{fi} \curvearrowright K]}_{[LT \curvearrowright [SL \curvearrowright K] \ LT \curvearrowright [\operatorname{if} OL \operatorname{fi} \curvearrowright K]]} & \operatorname{load-if2:} \underbrace{LT \curvearrowright [\operatorname{if} \ (:: SL) \ \operatorname{fi} \curvearrowright K]}_{[LT \curvearrowright [SL \curvearrowright K]]} \\ & \operatorname{load-goto:} LT \curvearrowright [\underbrace{.K}_{\operatorname{true}} \curvearrowright \operatorname{goto} X \curvearrowright K] & \operatorname{load-break:} LT \curvearrowright [\underbrace{.K}_{\operatorname{true}} \curvearrowright \operatorname{break} \curvearrowright K] \\ & \operatorname{true} \end{aligned}$$

Loading begins by lifting Loadable statements (i.e. basic and if statements) in a k cell into multisets, with a special loader token #load. #load (bound to the metavariable LT) guides the local rewrite within the multiset, by recursively decomposing each inner options until a leading basic statement is reached.

4.2 Examples

Our LOAD-AND-FIRE semantics handle nested options and cross-process interference in an elegant way, as illustrated by the running examples below.

Example 1. The nested if appearing in Figure 6 is loaded so that the basic statements A, B, and C appear upfront, exposing each branch's enabledness for firing (via fire/hs-fire). We underline the portion reduced by the loading rules:

```
\begin{split} &\langle \text{if} :: A :: \text{do} :: B :: C \text{ od } \text{fi}; D \rangle_k \\ &\rightarrow \langle \text{if} :: A :: \text{do} :: B :: C \text{ od } \text{fi} \cap D \rangle_k \\ &\rightarrow \langle \text{\#load} \cap [\text{if} :: A :: \text{do} :: B :: C \text{ od } \text{fi}] \cap D \rangle_k \\ &\rightarrow \langle [\text{\#load} \cap [A] \mid \text{\#load} \cap [\text{if} :: \text{do} :: B :: C \text{ od } \text{fi}]] \cap D \rangle_k \\ &\rightarrow \langle [\text{\#load} \cap [A] \mid \text{\#load} \cap [\text{do} :: B :: C \text{ od}]] \cap D \rangle_k \\ &\rightarrow \langle [\text{\#load} \cap [A] \mid \text{\#load} \cap [\text{if} :: B :: C \text{ fi} \cap \text{do} :: B :: C \text{ od}]] \cap D \rangle_k \\ &\rightarrow \langle [\text{\#load} \cap [A] \mid \text{\#load} \cap [\text{B} \cap \text{do} :: B :: C \text{ od}] \mid \text{\#load} \cap [\text{if} :: C \text{ fi} \cap \text{do} :: B :: C \text{ od}]] \cap D \rangle_k \\ &\rightarrow \langle [\text{\#load} \cap [A] \mid \text{\#load} \cap [B \cap \text{do} :: B :: C \text{ od}] \mid \text{\#load} \cap [C \cap \text{do} :: B :: C \text{ od}]] \cap D \rangle_k \\ &\rightarrow \langle [A \mid \text{\#load} \cap [B \cap \text{do} :: B :: C \text{ od}] \mid \text{\#load} \cap [C \cap \text{do} :: B :: C \text{ od}]] \cap D \rangle_k \\ &\rightarrow \langle [A \mid B \cap \text{do} :: B :: C \text{ od} \mid \text{\#load} \cap [C \cap \text{do} :: B :: C \text{ od}]] \cap D \rangle_k \\ &\rightarrow \langle [A \mid B \cap \text{do} :: B :: C \text{ od} \mid \text{\#load} \cap [C \cap \text{do} :: B :: C \text{ od}]] \cap D \rangle_k \end{split}
```

Example 2. Cross-process interference in Figure 7 is resolved in the fully loaded form (intermediate loading steps omitted):

$$\begin{array}{l} \langle \text{if} :: \text{c} : \text{1} :: \text{y} = \text{1} \; \text{fi} \rangle_{k_{I}} \; \langle \text{if} :: \text{y} = \text{1} :: \text{d} \; ? \; \text{x} \; \text{fi} \rangle_{k_{2}} \\ \rightarrow ! \langle [\text{c} : \text{1} \mid \text{y} = \text{1}] \rangle_{k_{I}} \langle [\text{d} \; ? \; \text{x} \mid \text{y} = \text{1}] \rangle_{k_{2}} \end{array}$$

Here, k_1 and k_2 denote (by abuse of notation) the k cells of p1 and p2, respectively. The rule hs-fire applies to this loaded form, making a joint nondeterministic choice across the two processes and thus capturing the interference.

5 More Extensions

In this section, we further extend the LOAD-AND-FIRE to incrementally add atomic blocks and run-expressions to our syntax, in a modular way. We briefly present the added/revised rules.

5.1 Adding Atomic Blocks

We add atomic blocks to our syntax (i.e., we drop BASIC GRANULARITY). Sequences in an atomic block execute atomically; e.g., atomic{ x++; x++; } is equivalent to x = x + 2. Intermixing atomic with constructs such as goto and handshake introduces subtle control-flow issues (Figure 8, 9) to be elaborated below. We outline extensions to the relevant \mathbb{K} rules that cover these cases.

```
active proctype p() {
  goto L;
  atomic {
    A; L: B
  }
}
```

```
chan q = [0] of { bool };
active proctype p1() {
  atomic { A; q!0; B }
}
active proctype p2() {
  atomic { q?0 ; C }
}
```

Fig. 8: Goto under atomicity.

Fig. 9: Handshake under atomicity.

Loading Rules. We modularly extend the loading rules to handle atomicity. Since if, do, and atomic can be nested in arbitrary combinations, atomic blocks may hide leading basic statements and must therefore be decomposed by loading. We present the extended portion, treating atomic blocks as *Loadable*. The old load-lift and load-bs are deprecated; all other rules remain unchanged.

```
\begin{aligned} & \operatorname{load-lift:} \ \langle P \rangle_{pid} \ \langle \underbrace{S}_{\# load(P = L) \ \curvearrowright [S]} \curvearrowright \ldots \rangle_k \ \langle L \rangle_{lock} & \text{if } Loadable(S) \end{aligned} & \operatorname{load-bs1:} \frac{\# load(\top)}{.K} \curvearrowright \underbrace{[BS \curvearrowright K]}_{\text{load-bs2:}} \frac{\# load(\bot)}{.K} \curvearrowright \underbrace{[BS \curvearrowright .K \curvearrowright K]}_{\# rel}  & \operatorname{load-at1:} \# load(\top) \curvearrowright \underbrace{[\operatorname{atomic} \ \{ \ SL \ \} \curvearrowright K]}_{SL} \longrightarrow K]  & \operatorname{load-at2:} \# load(\bot) \curvearrowright \underbrace{[\operatorname{atomic} \ \{ \ SL \ \} \curvearrowright K]}_{SL \curvearrowright \# rel} \longrightarrow K]
```

The purpose of these rules is to insert a single **#rel** at the end of the *outermost* atomic block, thereby enforcing atomic execution within it. We refine the loader token with a Boolean flag (written as \top/\bot inside of **#load(·)**) that records whether **#rel** has already been inserted to prevent incorrect early release. Together with the other loading rules, **#load(·)** is propagated along the nested structure, inserting **#rel** at each valid release points.

Firing Rules. We lift the firing rules to operate under atomicity, in accordance with the extended loading rules. The revised firing rules (i) no longer inserts #rel, as it is inserted by loading rules; and (ii) also matches a self-acquired lock, since basic statements may fire consecutively within an atomic block without releasing the lock.

```
\begin{array}{l} \text{fire:} \begin{array}{l} \langle P \rangle_{pid} \; \langle \underbrace{[BS \curvearrowright K_{local} \mid K_{rest}]}_{effect(BS) \curvearrowright K_{local}} \curvearrowright \ldots \rangle_k \; \langle ENV \rangle_{env} \; \langle STR \rangle_{str} \; \langle NET \rangle_{net} \; \langle \underline{L} \rangle_{lock} \\ & P \end{array} \\ & \text{if} \; L \in \{\bot, P\} \; \land \; \llbracket enabled(BS) \rrbracket_{ENV,STR,NET} = \top \end{array}
```

$$\mathsf{hs\text{-}fire:} \begin{array}{c} \langle ... \langle P \rangle_{pid} \ \langle \underbrace{[X \, [E] \ ! \ EL \curvearrowright K_{local} \mid K_{rest}]}_{[\mathsf{true} \curvearrowright K_{local}]} \curvearrowright ... \rangle_k \ \langle ENV \rangle_{env} \ ... \rangle_{proc} \ \langle STR \rangle_{str} \ \langle NET \rangle_{net} \\ \langle ... \langle P' \rangle_{pid} \ \langle \underbrace{[X' \, [E'] \ ? \ AL \curvearrowright K'_{local} \mid K'_{rest}]}_{(L = P \ ? \ \mathsf{\#toss}(P) \ : .K)} \curvearrowright \mathsf{\#assignMsg}(AL, EL) \curvearrowright K'_{local} \\ \langle \underbrace{L} \rangle_{lock} \qquad \qquad \mathsf{if} \ \underbrace{[X \, [E]]}_{ENV, STR, NET} = \mathsf{HChan} \ \underbrace{[X' \, [E']]}_{ENV', STR, NET} \\ \land \ L \in \{\bot, P, P'\} \ \land \ AL = \underbrace{[EL]}_{ENV, STR, NET} \end{array}$$

Additionally, under atomicity, handshake operations chained across multiple processes induce a global atomic chain of executions, as shown in Figure 9. If a handshake occurs while p1 is executing atomically, atomicity is transferred to p2, which executes block C; upon completion, atomicity returns to p1. This mechanism is implemented in hs-fire via lock tossing: if the sender already holds the lock (L=P), the lock is tossed back to the sender upon releasing the lock. The relevant rules for lock-tossing appear below.

Execution Rules. We list the execution rules updated for atomicity. Irrelevant execution rules from Section 3 remain unchanged.

$$\mathsf{toss1:} \ \frac{\langle \mathsf{\#toss}(P) \curvearrowright K}{K \curvearrowright \mathsf{\#toss}(P)} \curvearrowright \mathsf{\#rel} \curvearrowright \ldots \rangle_k \quad \mathsf{toss2:} \ \frac{\langle \mathsf{\#toss}(P) \curvearrowright \mathsf{\#rel} \curvearrowright \ldots \rangle_k \ \langle \cdot \rangle_{lock}}{K}$$

$$\mathsf{loose:} \ \frac{\langle P \rangle_{pid} \ \langle \ldots \curvearrowright}{(KI = \mathsf{\#toss}(\cdot) \ ? \ .K : KI)} \curvearrowright \mathsf{\#rel} \ldots \rangle_k \ \langle P \rangle_{lock} \quad \mathsf{otherwise}$$

$$\mathsf{goto:} \ \frac{\langle \ldots \langle P \rangle_{pid} \ \langle X \rangle_{pName} \ \langle \mathsf{goto} \ Y \ \curvearrowright \ldots \rangle_k \ldots \rangle_{proc} \ \langle \ L \ \rangle_{lock}}{K} \quad \mathsf{otherwise}$$

$$\langle \ldots \langle X \rangle_{ptName} \ \langle \ldots Y \mapsto (B, K) \ldots \rangle_{gotoMap} \ldots \rangle_{ptype}$$

The rules toss1 and toss2 implement lock tossing used in hs-fire. toss1 propagates the marker #toss(P) (with P the sender's pid) through the continuation until #rel. toss2 then sets the lock to P instead of releasing it.

When execution blocks inside an atomic block, atomicity is lost temporarily. The loose rule implements this by releasing the lock when no (firing) rules match, via the side condition "otherwise". Any pending toss marker, if present, is invalidated. Note that #rel still remains, preserving the original atomicity.

Finally, atomicity may switch abruptly via goto when the atomicity of the source and destination differs (Figure 8). The revised goto rule reflects the atomicity of the destination Y: if Y is (resp., is not) within an atomic block, indicated by $B = \top$ (resp., $B = \bot$), the lock is preserved (resp., released). Accordingly, the gotoMap is augmented with atomicity flags B for each label.

5.2 Adding Impure Expressions

As our final language extension, we add run expressions (i.e., we drop Pure Expressions). For simplicity, we only consider run expressions used in condition statements, and in the righthand-side of the assignment.

Evaluating run expressions incurs side effects of spawning new processes. For example, assuming the nextPid cell contains 42, the side effect of $X = run \ p1() + run \ p2()$ is to spawn two processes p1 and p2 and to assign X = 42 + 43. To reflect this, we extend the function $effect: BStmt \times Int \to K$ to be parameterized by nextPid as the second argument:

```
effect(E, I) = \#run(E)

effect(X[E] = E', I) = \#run(E') \curvearrowright \#assign(X, E, E' \upharpoonright_I)

effect(E, I) = effect(E) (otherwise, define as before)
```

, where the purification function $(\cdot) \upharpoonright_{(\cdot)}: BStmt \times Int \to BStmt$ substitutes all occurrences of **run** expressions in BS by the given pid's.

Accordingly, the fire rule is revised to use the updated effect and enabledness for basic statements that may involve run as subexpressions:

fire:
$$\frac{\langle P \rangle_{pid}}{effect(BS,P') \curvearrowright K_{local}} \stackrel{\frown}{\cap} \dots \rangle_k \langle ENV \rangle_{env} \langle STR \rangle_{str} \langle NET \rangle_{net}$$

$$\frac{\langle \bot \rangle_{lock}}{P} \stackrel{\frown}{\wedge} \gamma_{nextPid} \quad \text{if } L \in \{\bot,P\} \land [[enabled(BS \upharpoonright_{P'})]]_{ENV,STR,NET} = \top$$

For execution rules, we add rules that process the $\#\text{run}(\cdot)$ markers. These rules decompose #run(E) into #run(run X(EL))'s for each #run(EL) contained in E. The #run(EL) and #run(EL) contained in E.

```
run: \frac{\langle \# \text{run}(\text{run } X (EL)) }{\# \text{wait}} \curvearrowright ... \rangle_k \ \langle ENV \rangle_{env} \ \langle XS \rangle_{lVars} \\ \langle ... \ \langle X \rangle_{ptName} \ \langle DL \rangle_{params} \ \langle SL \rangle_{code} ... \rangle_{ptype} \ \langle \underbrace{P}_{P+1} \rangle_{nextPid} \\ \underline{Cell} \\ \overline{\langle \langle P \rangle_{pid} \ \langle X \rangle_{pName} \ \langle \# \text{init}(DL, EL) \ \curvearrowright \ \# \text{done} \ \curvearrowright SL \rangle_k \ \langle ENV \setminus XS \rangle_{env} \ \langle \emptyset \rangle_{local} \rangle_{proc}}
```

Auxiliary rules for the run rule can be found in Appendix B.

6 Case Study: Application to Deductive Verification

We implemented a prototype⁴ \mathbb{K} semantics of PROMELA. We validated our semantics via several examples with respect to the SPIN implementation. Using this prototype, we present a case study of using the \mathbb{K} deductive verifier to prove reachability claims for PROMELA programs under our \mathbb{K} semantics. Our case study includes examples for which SPIN fails to verify.

As a simple case, we revisit the reachability spec (Fig. 1) discussed in Section 2. Following the similar approach there, we can also prove the same claim for the counterpart program written in PROMELA shown in Figure 10. Note that leaving N as a parameter in the code yields an infinite family of models: each fixed N has a finite reachable state space (checkable by SPIN), but SPIN cannot verify the property for arbitrary N.

⁴ The semantics definition, along with the case study and benchmark examples for validation, are available at https://tinyurl.com/vmcai26-k-promela

```
int n = N, s = 0 // N is parameter
active proctype sum() {
   do
     :: 0 < n -> s = s + n ; n--
     :: 0 >= n -> break
   od
}
```

Fig. 10: A PROMELA code for summation.

```
int disp = 0, serv = 0, crit = 0;
                                                 active proctype p2() {
active proctype p1() {
                                                   int tick:
 int tick;
                                                   do
                                                     :: atomic { tick = disp:
 do
   :: atomic { tick = disp;
                                                                 disp = disp + 1 }
                                                       ; atomic { tick == serv;
                disp = disp + 1 }
      : atomic { tick == serv:
                                                                  crit = crit + 1
                 crit = crit + 1 }
                                                       : atomic { serv = serv + 1:
      ; atomic { serv = serv + 1;
                                                                  crit = crit - 1 }
                 crit = crit - 1 }
                                                  od
 od
                                                 active proctype monitor() { freeze }
}
```

Fig. 11: A PROMELA code for bakery algorithm with 2 processes with a monitor.

As a nontrivial example, we verify Lamport's Bakery algorithm [14] with two processes (having an infinite state space) for which SPIN cannot verify. Figure 11 presents the PROMELA program for the Bakery Algorithm involving two concurrent processes p1 and p2. Analogous to a real bakery (or bank), each process repeatedly: (i) obtains a ticket (tick) from the dispenser (disp), (ii) enters the critical section when the server (serv) calls its ticket, and then (iii) exits, via LOOP := do ... od. Each stage is denoted by the code fragment:

```
WAIT := atomic { tick = disp; disp = disp + 1 }; ENTER
ENTER := atomic { tick == serv; crit = crit + 1 }; EXIT
EXIT := atomic { serv = serv + 1; crit = crit - 1 }
```

Note that tick can grow indefinitely, inducing an infinite number of states.

We verify mutual exclusion (mutex) between p1 and p2, by asserting the invariant $\mathtt{crit} \leq 1$ for all reachable states. To express this property for nonterminating systems in reachability logic, we follow the similar approach proposed in [32]: we introduce a monitor process that nondeterministically "freezes" the global state via a pseudo-statement freeze, by setting the lock to a special value #frozen (also denoted " \circledast "). By defining the goal pattern ϕ_{goal} as the set of "frozen" states where $\mathtt{crit} \leq 1$ holds, the main claim $\phi_{init} \Rightarrow \phi_{goal}$ asserts mutex for all finite prefix of the original bakery code without monitor, from the initial pattern ϕ_{init} with $\mathtt{disp=serv}=N$ for some integer N.

Figure 12 shows the concrete spec for our main claim. We succintly write this main claim in the following high-level notation:

```
\langle LOOP, LOOP, freeze, N, N, O, \bot \rangle \Rightarrow \langle ?\_, ?\_, .K, ?\_, ?\_, ?C \leq 1, \circledast \rangle (1)
```

```
claim:
                                                          claim:
2
      cs>
                                                      2
                                                            cs>
         // proc p1
  <k> LOOP => ?_ </k>
                                                              3
                                                      3
4
                                                      4
5
          <env>
                                                      5
                                                                <env>
      disp[0] \rightarrow loc(0): serv[0] \rightarrow loc(1)
                                                            disp[0] |-> loc(0); serv[0] |-> loc(1)
6
                                                      6
      crit[0] |-> loc(2); tick[0] |-> loc(3)
                                                            crit[0] |-> loc(2); tick[0] |-> loc(3)
                                                      7
          </env>
                                                      8
                                                                </env>
     ...</proc>
                                                      9
                                                              </proc>
        >... // proc p2
                                                              >... // proc p2
          < k > LOOP => ?_ </ k >
                                                                <k> WAIT! => ?_ </k>
          <env>
                                                                <env>
      disp[0] |-> loc(0); serv[0] |-> loc(1)
                                                            disp[0] |-> loc(0); serv[0] |-> loc(1)
13
                                                     13
14
      crit[0] \rightarrow loc(2); tick[0] \rightarrow loc(4)
                                                     14
                                                            crit[0] \rightarrow loc(2); tick[0] \rightarrow loc(4)
15
          </env>
                                                     15
                                                                </env>
16
     ...</proc>
                                                     16
                                                           ...</proc>
                                                               // proc monitor
17
        <>... // proc monitor
                                                     17
18
          \langle k \rangle freeze => .K \langle k \rangle
                                                     18
                                                                \langle k \rangle freeze => .K \langle k \rangle
                                                              </proc>
19
       ..</proc>
                                                     19
      </procs>
                                                            </procs>
20
                                                     20
21
      <str>
                                                            <str>
                                                     21
                                                              0 |-> (N +Int 1 => ?_);
        0 |-> (N:Int => ?_); 1 |-> (N => ?_);
        2 |-> (0 => ?C);
                                                     23
                                                              1 |-> (N:Int => ?_); 2 |-> (0 => ?C);
        3 |-> (0 => ?_); 4 |-> (0 => ?_)
                                                              3 \mid -> (N \Rightarrow ?_); 4 \mid -> (0 \Rightarrow ?_)
                                                     24
      </str>
                                                            </str>
      <lock>#none => #frozen </lock>
                                                     26
                                                            <lock> #none => #frozen </lock>
     ensures ?C <=Int 1 // MUTEX
                                                          ensures ?C <=Int 1 // MUTEX
```

Fig. 12: The main claim

Fig. 13: An auxiliary claim

The seven components in the tuple correspond to: (the continuations of) p1,p2, monitor, (the values of) disp (line 22), serv (line 22), crit (line 23), and the global lock (line 26). In the goal pattern, "?_" denote "don't care" variables, and "?C \leq 1" means crit \leq 1, where "?" indicate newly introduced variables.

During the proof of the main claim, we found that the bakery code induces three cycles that do not contain ϕ_{init} . As noted in Section 2, they can be ignored via adding auxiliary claims, listed as follows:

```
 \langle \mathtt{WAIT!}(\mathtt{T}), \mathtt{WAIT!}(\mathtt{T}), \mathtt{freeze}, \mathtt{N}, \mathtt{N}, \mathtt{0}, \bot \rangle \Rightarrow \langle ?\_, ?\_, \mathtt{K}, ?\_, ?\_, ?\mathtt{C} \leq \mathtt{1}, \circledast \rangle  (2)  \langle \mathtt{ENTER!}(\mathtt{N}), \mathtt{WAIT!}(\cdot), \mathtt{freeze}, \mathtt{N} + \mathtt{1}, \mathtt{N}, \mathtt{0}, \bot \rangle \Rightarrow \langle ?\_, ?\_, \mathtt{K}, ?\_, ?\_, ?\mathtt{C} \leq \mathtt{1}, \circledast \rangle  (3)  \langle \mathtt{WAIT!}(\cdot), \mathtt{ENTER!}(\mathtt{N}), \mathtt{freeze}, \mathtt{N} + \mathtt{1}, \mathtt{N}, \mathtt{0}, \bot \rangle \Rightarrow \langle ?\_, ?\_, \mathtt{K}, ?\_, ?\_, ?\mathtt{C} \leq \mathtt{1}, \circledast \rangle  (4)
```

Here, WAIT!(T) denotes the fully-loaded continuation for WAIT, under the local variable tick=T, and likewise for ENTER!(T). Indeed, the initial pattern of the claim 2 is simply the fully-loaded version of the initial pattern of the claim 1. The initial patterns of the claims 3 and 4 are obtained by executing the atomic block of p1 and p2, respectively, from the initial pattern of the claim 2. Figure 13 shows the concrete spec for the claim 3.

Encoding the four claims (1 main / 3 auxiliary) in spec.k and running it with kprove, it successfully terminates with output #Top in approximately 5 minutes in a laptop computer (i5-1335U 2.50 GHz / 16 GB RAM).

7 Related Work

Substantial work exists on establishing a formal semantics of PROMELA. In [23,2,24], the semantics is given by a low-level labeled transition system. [38,17,31] give structural operational semantics (SOS) [28], while [18] gives a denotational semantics. From the analysis perspective, [38,24] remain in the scope of SPIN's LTL verification. Other lines of work extend the analysis to other techniques such as abstract interpretation [17,18], and PROMELA-to-C refinement [31]. We identified two works that establish mechanized, executable semantics of PROMELA—one in ACL2 [2] and another in Isabelle/HOL [24]. However, to the best of our knowledge, no prior work provides an executable semantics that enables code-level deductive verification of PROMELA.

 \mathbb{K} [30] is an executable semantic framework for defining programming languages. It emphasizes modularity—addressing a well-known limitation of SOS—so new features can be added without revising unrelated rules. \mathbb{K} has proved effective for control-intensive features (e.g., exceptions, call/cc) and has been used to formalize several real-world languages [9,3,25,37]. The framework was later formalized as Matching Logic [29], yielding a deductive system [6] with commercial applications, notably in smart-contract verification [27,26,11]. This work presents the first executable PROMELA semantics defined in \mathbb{K} .

K is grounded in Rewriting Logic [20], which is introduced as a general formalism for specifying concurrent systems [19]. Classic process calculi—including CCS [21] and the Pi-calculus [22]— have been studied within Rewriting Logic [4,7,35,36,33,34]. Rewriting Logic's inherent nondeterminism and true concurrency make such semantics natural to specify; we exploit this in our LOAD-AND-FIRE semantics via multiset matching.

8 Concluding Remarks

We have presented a faithful, executable semantics of PROMELA in the K framework. Our semantics enables code-level deductive verification of PROMELA models, including infinite-state systems, a capability previously unavailable for PROMELA. Beyond providing a precise, machine-readable reference, it bridges model checking and deductive reasoning: properties beyond explicit-state model checking can now be proved directly on PROMELA programs.

We have also introduced LOAD-AND-FIRE, an elegant semantic pattern that yields a modular, uniform treatment of guarded nondeterminism, cross-process interference, and atomicity in \mathbb{K} . Beyond PROMELA, this constitutes a reusable methodology for \mathbb{K} -based language semantics of guarded concurrency, with natural applications to constructs such as Go's select and Erlang's receive.

Future work includes supporting temporal reasoning for deductive verification (e.g., LTL) strengthening proof automation for PROMELA (e.g., reusable lemma libraries), and conducting additional case studies on complex PROMELA models. We also plan to apply LOAD-AND-FIRE to other programming languages with guarded choice and synchronous communication.

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Appendix

Rules for Local Declaration

The statement T[X] declares a local (array) variable X of (integer) size I, initialized by default values defined for each typename $T \in \{\text{int}, \text{bool}, \text{chan}\}$.

Declaration for primitive types is defined by the decl-prim rule, where str is initialized to the default value initVal(PT) for $PT \in \{int, bool\}$:

decl-prim:

We write $M[V_1, ..., V_n/X_0, ..., X_n]$ to denote parallel substitution for a map M. In the rule, the environment binds each indexed variable to location $loc(\cdot)$.

For channel declarations, the variables are directly bound to channel objects. The decl-uch rule for uninitialized channel declaration binds the variable to the undefined channel uch.

$$\texttt{decl-uch: } \frac{\langle \texttt{chan} \ X \, [I]}{.K} \curvearrowright ... \rangle_k \ \langle \frac{ENV}{ENV[uch/X[0], ..., uch/X[I-1]]} \rangle_{env} \ \langle \frac{XS}{XS \cup \{X\}} \rangle_{lVars}$$

For initialized channel declarations, The rules decl-bch and decl-hch binds each variable to a buffered channel $bch(\cdot)$ and handshake channel $hch(\cdot)$, resp.

decl-bch:

decl-hch:

$$\begin{array}{l} \langle \underline{\text{chan } X[I] = [0] \text{ of } \{TL\}} \curvearrowright \ldots \rangle_k \ \langle \underbrace{ENV[hch(J)/X[0], \ldots, hch(J+I-1)/X[I-1]]}^{ENV} \\ \langle \underbrace{XS}_{XS \ \cup \ \{X\}} \rangle_{lVars} \ \langle \underbrace{J}_{J+I} \rangle_{nextHCid} \end{array}$$

B Rules for Run Expressions

A composite run expression is decomposed via the rules run1 – run3.

$$\operatorname{run1:} \frac{\langle \operatorname{\#run}(E \odot E') \\ \neg \operatorname{\#run}(E) \\ \neg \operatorname{\#run}(E')}{}^{\frown} \dots \rangle_k}{\operatorname{\#run}(E)} \operatorname{run2:} \frac{\langle \operatorname{\#run}(\ominus E) \\ \neg \dots \rangle_k}{\operatorname{\#run}(E)}$$

$$\operatorname{run3:} \frac{\langle \operatorname{\#run}(E) \\ \neg \dots \rangle_k}{K} \quad \text{otherwise}$$

The rules sync,init1, and init2 are used in the main run rule to initialize a spawned process in a synchronized way.

$$\mathrm{sync} \colon \frac{\langle \mathtt{\#wait} \curvearrowright ... \rangle_k}{.K} \, \frac{\langle \mathtt{\#done}}{.K} \curvearrowright ... \rangle_k \quad \mathrm{init1} \colon \frac{\langle \mathtt{\#init}(\mathit{nil}, \mathit{nil}) \curvearrowright ... \rangle_k}{.K}$$

$$\texttt{init2:} \langle \underbrace{K}_{T \ X[E] \ \curvearrowright \texttt{\#assign}(X, E, E')} \\ \curvearrowright \texttt{\#init}(\underbrace{T \ X[E]}_{nil}, DL, \underbrace{E'}_{nil}, EL) \\ \smallfrown \ldots \rangle_k$$